CHAPTER 7 WORK AND ENERGY

Select odd-numbered solutions, marked with a dagger (†), appear in the Student Solutions Manual, available for purchase. Answers to all solutions below are underscored.

†7-1. The work done by a force F is $W = Fs\cos\theta$, where s is the displacement and the angle θ is the angle between the force and the displacement vectors. Since the force applied to the car is in the same direction as the displacement, the angle is 0° and cos 0° = 1.
$W = Fs\cos\theta = (300\text{ N})(5.0\text{ m})\cos 0° = 1500\text{ J} = 1.5 \times 10^3\text{ J}$

7-2. During the lift, the force exerted by the weight lifter must be directed upward and is equal in magnitude to the downward force of gravity on the barbell, assuming the lift is done at constant velocity. The magnitude of the weight of the barbell is mg. Therefore, the work done is
$W = F_y\Delta y = mg\Delta y = (254\text{ kg})(9.81\text{ m/s}^2)(1.98\text{ m}) = 4930\text{ J} = 4.93 \times 10^3\text{ J}$.
Since the force applied by the weight lifter is in the same direction as the barbell is displaced, the work done on the barbell is positive.

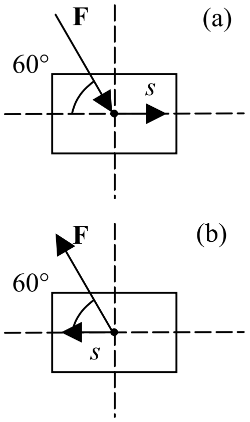
†7-3. With each movement, either forward or backward, the displacement and the force are in the same direction. Therefore, positive work is done on the saw.
$W = F_x\Delta x = (35\text{ N})[(30 \times 0.12\text{ m}) + (30 \times 0.12\text{ m})] = 252\text{ J}$

7-4. The work done on the bucket to raise it is $W = F_y\Delta y$, where the upward force on the bucket is F_y and it is raised a distance Δy from the bottom of the well to the top. Assuming the bucket is raised at a constant speed, the magnitude of the upward force is equal to the weight mg of the bucket. Using the definition of work, solve for Δy.

$$\Delta y = \frac{W}{F_y} = \frac{2200\text{ J}}{(15\text{ kg})(9.81\text{ m/s}^2)} = 15\text{ m}$$

7-5. $W = F_x\Delta x = f_{k,1}\Delta x_1 + f_{k,2}\Delta x_2 = N[\mu_{k,1}\Delta x_1 + \mu_{k,2}\Delta x_2] = mg[\mu_{k,1}\Delta x_1 + \mu_{k,2}\Delta x_2]$
$= (20\text{ kg})(9.81\text{ m/s2})[(0.25)(10\text{ m}) + (0.55)(30\text{ m})] = 3700\text{ J}$

7-6. The forward and backward motions are represented in parts (a) and (b) of the figures, respectively. In both cases, the horizontal component of the force is in the direction of the displacement and positive work is done on the vacuum. The total work is then
$W = Fs\cos\theta = 300\,(40\text{ N})(1.0\text{ m})\cos 60° = 6000\text{ J}$.



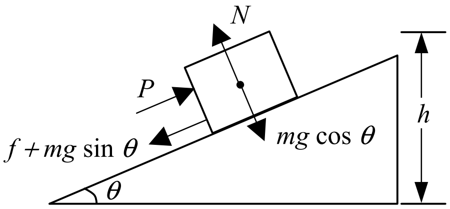
†7-7. $W = F\Delta y = mg\Delta y = (75\text{ kg})(9.81\text{ m/s}^2)(320\text{ m}) = 235\,000\text{ J}$
The average rate of doing work is

$$\frac{W}{\Delta t} = \frac{235\,000\text{ J}}{659\text{ s}} = 357\text{ J/s}.$$

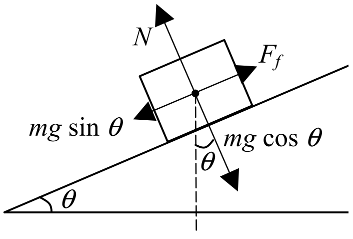
7-8. $W = Fs\cos\theta = (50\text{ N})(1.6\text{ m})\cos 60° = 40\text{ J}$

†7-9. The first tugboat does positive work W_1 on the barge, where
$W_1 = F_1 s\cos\theta_1 = (2.5 \times 10^5\text{ N})(100\text{ m})\cos 30° = 2.2 \times 10^7\text{ J}$.
Similarly, the second boat does positive work W_2 on the barge, where
$W_2 = F_2 s\cos\theta_2 = (1.0 \times 10^5\text{ N})(100\text{ m})\cos 15° = 1.0 \times 10^7\text{ J}$.
The total work done by both tugboats is then $W = W_1 + W_2 = 3.2 \times 10^7\text{ J}$.

7-10. $W_g = F_g\Delta y = -mg\Delta y = -(2.0\text{ kg})(9.81\text{ m/s}^2)(4.0\text{ m}) = -78\text{ J}$



†7-11. P is the push applied by the man. From the free-body diagram, $N = mg\cos\theta$ and $P = mg\cos\theta + f$, where $f = \mu_k N = \mu_k mg\cos\theta$ is the frictional force. Therefore, $P = mg\cos\theta + \mu_k mg\cos\theta$. To raise the box to a height h, the box is pushed up the incline a distance $s = h/\sin\theta$. The work done on the box by the man is $W = Ps = (mg\cos\theta + \mu_k mg\cos\theta)(h/\sin\theta)$, which may be simplified to $W = mgh(1 + \mu_k\cot\theta) = (60\text{ kg})(9.81\text{ m/s}^2)(2.5\text{ m})[1 + (0.45)\cot 30°] = \underline{2.6 \times 10^3\text{ J}}$.



7-12. The free-body diagram shows the three forces acting on the car as it coasts downhill: the normal force N, the gravitational force mg, and the combined frictional forces F_f. The angle θ is found from the given slope, $\theta = \tan^{-1}(1/20)$. Since the car coasts at constant speed, the magnitude of the frictional force is equal to the component of the gravitational force directed down the incline, $F_f = mg\sin\theta = (1200\text{ kg})(9.81\text{ m/s}^2)\sin[\tan^{-1}(1/20)] = \underline{590\text{ N}}$. This assumes that the mass of the driver is included in the 1200 kg. The work done by the frictional forces is $W = F_f s = (-590\text{ N})(1000\text{ m}) = \underline{-5.9 \times 10^5\text{ J}}$.

7-13. $W = Fgs = (mg\sin\theta)s = (1700\text{ kg})(9.81\text{ m/s}^2)(\sin 25°)(10\text{ m}) = \underline{7 \times 10^4\text{ J}}$
The work done by the frictional force is -7×10^4 J, since the net work on the car is 0 J. Then $0\text{ J} = W_g + W_f + W_N = 7 \times 10^4\text{ J} + W_f + 0\text{ J}$ and $W_f = \underline{-7 \times 10^4\text{ J}}$

7-14. The frictional force due to air resistance is $f_{\text{air}} = \frac{1}{2}C\rho Av^2$, where the aerodynamic constant for the car is $C = 0.30$, ρ is the density of air, the cross-sectional area for this car is $A = 2.8\text{ m}^2$. Using this information, the work done on the car by air resistance is

$$W_{20} = f_{\text{air}}\Delta x = \tfrac{1}{2}C\rho Av^2\Delta x$$
$$= \tfrac{1}{2}(0.30)(1.3\text{ kg/m}^3)(2.8\text{ m}^3)(20\text{ m/s})^2(250\ 000\text{ m}) = \underline{5.5 \times 10^7\text{ J}}.$$

When the speed is increased to 30 m/s, the work done increases to

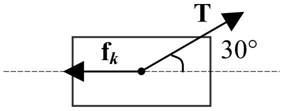
$$W_{30} = \tfrac{1}{2}(0.30)(1.3\text{ kg/m}^3)(2.8\text{ m}^3)(30\text{ m/s})^2(250\ 000\text{ m}) = \underline{1.2 \times 10^8\text{ J}}.$$

7-15. $\theta = \cos^{-1}(W/Fs) = \cos^{-1}[(175\text{ J})/(25\text{ N})(12\text{ m})] = \underline{54°}$

7-16. As the man walks north, the component of the wind force F that acts on him is $F\cos 30°$. When he walks east, the component is $F\sin 30°$. In both cases, positive work will be done. The total work is the sum of the work done during each segment.
$W = F_1 s_1 + F_2 s_2 = (F\cos 30°)s_1 + (F\sin 30°)s_2$
$= (150\text{ N})(\cos 30°)(100\text{ m}) + (150\text{ N})(\sin 30°)(200\text{ m}) = \underline{2.8 \times 10^4\text{ J}}$

†7-17. (a) If the cart is to move at constant velocity, the magnitude of the applied force T_1 must equal that of the kinetic frictional force f_k, which is oppositely directed. The work the man does on the cart is $W_1 = T_1\Delta x = (250\text{ N})(50\text{ m}) = \underline{1.3 \times 10^4\text{ J}}$.

(b) When the force is applied in the direction shown, work is done by the horizontal component of the tension **T**. Because the vertical component of the tension would reduce the normal force and also the frictional force.

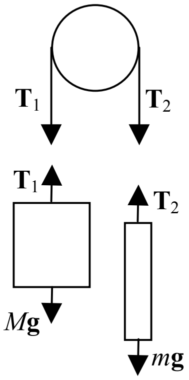


In this problem, additional mass has been added so that the frictional force is the same in both cases. To determine the tension, we assume that the resulting horizontal acceleration is equal to zero, so that $T_2\cos 30° - f_k = 0$. Solving for T gives,
$T_2 = f_k/(\cos 30°) = (250\text{ N})/(\cos 30°) = \underline{290\text{ N}}$.

The work the man must now do is
$W_2 = T(\cos 30°)\Delta x = (290\text{ N})(\cos 30°)(50\text{ m}) = \underline{1.3 \times 10^4\text{ J}}$, which is the same as W_1.

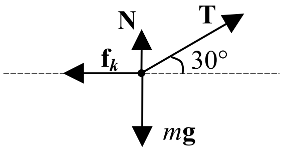
7-18. $W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy = \int_0^2 3dx + \int_0^{-1} 2dy = 3x\big|_0^2 + 2y\big|_0^{-1} = 6\text{ J} - 2\text{ J} = \underline{4\text{ J}}$

†7-19. (a)The net force on the elevator is $T_1 - Mg = Ma$, where T_1 is the tension in the elevator cable and M is the elevator mass including its load.
$T_1 = M(g + a) = (1200\text{ kg})(9.81\text{ m/s}^2 + 1.5\text{ m/s}^2) = \underline{1.4 \times 10^4\text{ N}}$
For the counterweight, the tension is
$T_2 = m(g - a) = (1000\text{ kg})(9.81\text{ m/s}^2 - 1.5\text{ m/s}^2) = \underline{8.3 \times 10^3\text{ N}}$.
(b)To determine the distance traveled in the 1.0 s interval, apply the kinematic relation,
$y = \frac{1}{2}at^2 = \frac{1}{2}(1.5\text{ m/s}^2)(1.0\text{ s}) = 0.75\text{ m}.$



Assuming the elevator is moving upward during the interval, the work done by the pulley motor is
$W = T_1y + T_2y = (1.4 \times 10^4\text{ N})(0.75\text{ m}) - (8.3 \times 10^3\text{ N})(0.75\text{ m}) = \underline{4300\text{ J}}$.
(c) For motion at constant speed, $T_1 = Mg = (1200\text{ kg})(9.81\text{ m/s}^2) = \underline{1.2 \times 10^4\text{ N}}$ and
$T_2 = mg = (1000\text{ kg})(9.81\text{ m/s}^2) = \underline{9.8 \times 10^3\text{ N}}$
The work done during the upward displacement is
$W = (T_1 - T_2)y = (1.2 \times 10^4 - 9.8 \times 10^3)(10.0\text{ m}) = \underline{2.2 \times 10^4\text{ J}}.$

7-20. Since the sled has no vertical acceleration, the vertical component of the net force on the sled can be used to find the magnitude of the normal force, $N = mg - T\sin 30°$. Since the sled moves with constant speed, the magnitude of the horizontal component of the tension equals that of the kinetic friction force, $f_k = T\cos 30°$.



By applying the relationship between the normal force and friction, the tension in the rope can be determined.

$$\mu_k N = T\cos 30°$$

$$\mu_k(mg - T\sin 30°) = T\cos 30°$$

Solving for the tension, gives

$$T = \frac{\mu_k mg}{\cos 30° + \mu_k \sin 30°} = \frac{(0.10)(150\text{ kg})(9.81\text{ m/s}^2)}{\cos 30° + (0.10)\sin 30°}(1000\text{ m}) = 160\text{ N}.$$

The work done by the girl on the sled is equal to the product of the horizontal component of the tension and the distance traveled,
$W = (T\cos 30°)\Delta x = (160\text{ N})(\cos 30°)(1000\text{ m}) = \underline{1.4 \times 10^5\text{ J}}$.

†7-21. (a) The angle the rope makes with respect to the horizontal direction is $\theta = \sin^{-1}(10\text{ m}/50\text{ m})$. Since the boat is stationary, the horizontal component of the tension must be equal in magnitude to the wind force. Thus, $T\cos\theta = 7000$ N, which can be solved for the tension

$$T = \frac{7000\text{ N}}{\cos\theta} = \frac{7000\text{ N}}{\cos\left[\sin^{-1}\left(\frac{10\text{ m}}{50\text{ m}}\right)\right]} = \underline{7100\text{ N}}.$$

(b) Since the wind force is 7000 N, the sailors must exert 7000 N in the direction of motion assuming the boat is to be pulled at constant speed. If 30 m of rope is pulled in, the hypotenuse is reduced to 20 m. The horizontal distance is found from the Pythagorean theorem,

$$x_2 = \sqrt{(20\text{ m})^2 - (10\text{ m})^2} = 17\text{ m}$$

Similarly, the original distance was

$$x_1 = \sqrt{(50\text{ m})^2 - (10\text{ m})^2} = 49\text{ m}$$

So, the net displacement of the boat is (49 m − 17 m) = 32 m and the work done is $W = (7000\text{ N})(32\text{ m}) = \underline{2.2 \times 10^5\text{ J}}$.

In this new position, the tension is

$$T = \frac{7000\text{ N}}{\cos\theta} = \frac{7000\text{ N}}{\cos\left[\sin^{-1}\left(\dfrac{10\text{ m}}{20\text{ m}}\right)\right]} = \underline{8100\text{ N}}.$$

7-22. To compress the spring 0.10 m, the work done on the spring is

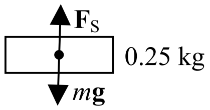
$$W = \tfrac{1}{2}k(x_2^2 - x_1^2) = \tfrac{1}{2}(3.5\times10^4\text{ N/m})[(0.10\text{ m})^2 - (0\text{ m})^2] = \underline{180\text{ J}}.$$

To compress the spring by another 0.10 m, requires additional work equal to

$$W = \tfrac{1}{2}k(x_2^2 - x_1^2) = \tfrac{1}{2}(3.5\times10^4\text{ N/m})[(0.20\text{ m})^2 - (0.10\text{ m})^2] = \underline{530\text{ J}}.$$

†7-23. The work is equal to the area under the F_x versus x curve. The work done in moving between 0 and 6 m is + 8 J. Between 6 and 8 m, the work done is −2 J. Therefore, the total work done is $W = +8\text{ J} - 2\text{ J} = \underline{+6\text{ J}}$.

7-24. The spring constant can be determined from the information given since the sum of the vertical forces is $k(\Delta y) - mg = 0$, $k = mg/(b-a)$, where a and b are the initial and final positions, respectively. The free body diagram shows the two forces acting on the object hung from the spring.



The work done by gravity on the object is

$$W_g = \int_a^b F_y(y)dy = \int_a^b -mg\,dy = -mg\int_a^b dy$$

$$= -mg(b-a) = -(0.25\text{ kg})(9.81\text{ m/s}^2)[(-0.18\text{ m}) - (0\text{ m})] = \underline{0.44\text{ J}}.$$

The work done by the spring is

$$W_S = \int_a^b F_y(y)dy = \int_a^b -ky\,dy = -\tfrac{1}{2}k(b^2 - a^2) = -\frac{mg}{2(b-a)}(b^2 - a^2)$$

$$= -\frac{(0.25\text{ kg})(9.81\text{ m/s}^2)}{2(-0.18\text{ m} - 0\text{ m})}[(-0.18\text{ m})^2 - (0\text{ m})^2] = \underline{0.22\text{ J}}.$$

†7-25. The spring force is a restoring force, which is directed so as to return the spring to its equilibrium position. In this problem, the mass attached to the end of the spring is displaced in the positive x direction, but as the mass passes $x = 0$ m, the spring force changes direction. Since the force is varying, the work done by the spring is found through integration.

$$W_S = \int_a^b F_x(x)dx = \int_{-0.20\text{ m}}^{0.40\text{ m}} -kx\,dx = -\tfrac{1}{2}kx^2\Big|_{-0.20\text{ m}}^{0.40\text{ m}}$$

$$= -\tfrac{1}{2}(440\text{ N/m})[(0.40\text{ m})^2 - (-0.20\text{ m})^2] = \underline{-26\text{ J}}$$

7-26. Someone applies a force on the spring to stretch it by 0.027 m, so positive work is done on the spring. The force is equal in magnitude to the restoring force of the spring, kx. Therefore, the work done on the spring is

$$W_S = \int_a^b F_x(x)dx = \int_0^{0.027\text{ m}} kx\,dx = \tfrac{1}{2}kx^2\Big|_0^{0.027\text{ m}}$$

$$= \tfrac{1}{2}(4500\text{ N/m})[(0.027\text{ m})^2 - (0\text{ m})^2] = \underline{1.6\text{ J}}.$$

†7-27. To stretch the spring from 0 to d, requires work $W_0 = -\frac{1}{2}kd^2$. The amount of work to further stretch the spring to $2d$ is

$$W = \int_d^{2d} F_x(x)dx = \int_d^{2d} -kx\,dx = -\tfrac{1}{2}kx^2\Big|_d^{2d}$$

$$= -\tfrac{1}{2}k[(2d)^2 - (d)^2] = \underline{-\tfrac{3}{2}kd^2 = 3W_0}.$$

If we now wish to generalize this for greater distances, the work done would be

$$W = \int_{Nd}^{(N+1)d} kx\,dx = \tfrac{1}{2}kx^2\Big|_{Nd}^{(N+1)d}$$

$$= -\tfrac{1}{2}k[(N+1)^2d^2 - (Nd)^2] = \underline{-\tfrac{1}{2}k(2N+1)d^2 = (2N+1)W_0}.$$

7-28. The work done by the non-ideal spring is

$$W_{NI} = \int_a^b F_x(x)dx = \int_a^b (6x - 2x^3)\,dx = 3x^2 - \tfrac{1}{2}x^4\Big|_a^b.$$

We will compare this with the work done by an ideal spring, which is $W_I = \int_a^b kx\,dx = \tfrac{1}{2}kx^2\Big|_a^b$.

For $x = 0$ to 0.50 m, $W_{NI} = 3x^2 - \tfrac{1}{2}x^4\Big|_0^{0.50\text{ m}} = 3(0.50\text{ m})^2 - \tfrac{1}{2}(0.50\text{ m})^4 = \underline{0.72\text{ J}}$.

and $W_I = \tfrac{1}{2}kx^2\Big|_0^{0.50\text{ m}} = \tfrac{1}{2}(6.0\text{ N/m})(0.50\text{ m})^2 = \underline{0.75\text{ J}}$.

For $x = 1.0$ to 1.50 m,

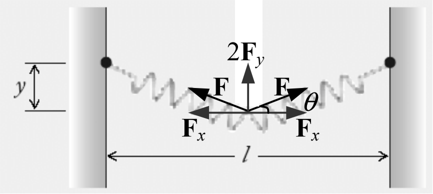
$W_{NI} = 3x^2 - \tfrac{1}{2}x^4\Big|_{1.0\text{ m}}^{1.5\text{ m}} = [3(1.5\text{ m})^2 - \tfrac{1}{2}(1.5\text{ m})^4] - [3(1.0\text{ m})^2 - \tfrac{1}{2}(1.0\text{ m})^4] = \underline{1.7\text{ J}}$

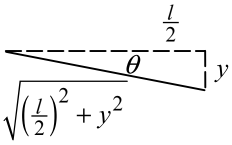
and $W_I = \tfrac{1}{2}kx^2\Big|_0^{0.50\text{ m}} = \tfrac{1}{2}(6.0\text{ N/m})[(1.5\text{ m})^2 - (1.0\text{ m})^2] = \underline{3.8\text{ J}}$.

For $x = 2.0$ to 2.50 m, $W_{NI} = [3(2.5\text{ m})^2 - \tfrac{1}{2}(2.5\text{ m})^4] - [3(2.0\text{ m})^2 - \tfrac{1}{2}(2.0\text{ m})^4] = \underline{3.2\text{ J}}$

and $W_I = \tfrac{1}{2}(6.0\text{ N/m})[(2.5\text{ m})^2 - (2.0\text{ m})^2] = \underline{6.8\text{ J}}$.

†7-29. In the vertical direction, the magnitude of the external force is equal to twice the vertical component of the restoring force of the spring F_y. Consider one-half of the spring as represented in the triangle below the figure to the right. The extension of one-half of the spring from its equilibrium length of $l/2$ is equal to $\Delta l = \sqrt{(l/2)^2 + y^2} - l/2$.





The applied force is

$$P = 2F\sin\theta = 2k(\Delta l)\frac{y}{\sqrt{(l/2)^2 + y^2}}.$$

where F is the magnitude of the force exerted by the spring. Substituting the equation for Δl gives

$$P = 2k\left[\sqrt{(l/2)^2 + y^2} - l/2\right]\frac{y}{\sqrt{(l/2)^2 + y^2}} = 2k\left[y - \frac{y(l/2)}{\sqrt{(l/2)^2 + y^2}}\right]$$

$$\underline{P = 2ky\left[1 - \frac{(l/2)}{\sqrt{(l/2)^2 + y^2}}\right]}.$$

The work done on the spring by the force *P* is

$$W=\int_0^y P\,dy=2k\int_0^y y\left[1-\frac{(l/2)}{\sqrt{(l/2)^2+y^2}}\right]dy$$

$$=2k\left\{\frac{y^2}{2}-\frac{l}{2}\left[\sqrt{(l/2)^2+y^2}-(l/2)\right]\right\}=\underline{k\left[y^2+\frac{l^2}{2}+\frac{l}{2}\sqrt{(l/2)^2+y^2}\right]}.$$

As a check, the work done by the external force on the spring in displacing it by a distance *y* and extending it by Δl is equal to the change in the spring potential energy, $W=\frac{1}{2}k(\Delta l)^2$.

Then, the total work done on the whole spring is twice this value,

$$\underline{W=2\left(\frac{1}{2}k(\Delta l)^2\right)=k\left[\sqrt{(l/2)^2+y^2}-l/2\right]^2}$$

which is the same result obtained when the quantity in the square brackets is expanded.

7-30. $W=\int_{x_1}^{x_2}F(x)\,dx=\int_0^2(2x^2+8x)\,dx=\frac{2}{3}x^3+4x^2\Big|_0^{2\text{ m}}=\underline{21\text{ J}}$

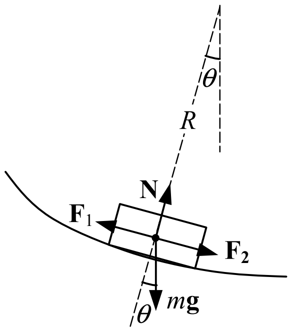
†7-31. Consider the path from the origin to the point (2 m, 2 m, 0). This is a straight line path on which $x=y$ and, therefore, $dx=dy$. The work done on this path is

$$W=\int_{x_1}^{x_2}F_x\,dx+\int_{y_1}^{y_2}F_y\,dy+\int_{z_1}^{z_2}F_z\,dz$$

$$=\int_0^{x_2}(4\text{x}^2+1)\,dx+\int_0^{y_2}2x\,dy=\int_0^{2\text{ m}}(4\text{x}^2+1)\,dx+\int_0^{2\text{ m}}2x\,dx$$

$$=\tfrac{4}{3}x^3+x+x^2\Big|_0^{2\text{ m}}=\underline{17\text{ J}}.$$

7-32. Consider the free body diagram for this situation. The horse is pulling upward along the arc with a force $\mathbf{F}_1$ for a distance *x* at constant speed. The magnitude of $\mathbf{F}_1$ is equal to the magnitude of $\mathbf{F}_2$, which is the sum of frictional force $f=\mu_k N$ and the component of the weight directed downward along the arc, $mg\sin\theta$. The magnitude of the normal force is equal to the component of the weight perpendicular to the arc, $N=mg\cos\theta$. Thus, $F_1=\mu_k\,mg\cos\theta+mg\sin\theta$. The work done by the horse is



$$W=\int_a^b F(x)\,dx=\int_0^x mg(\mu_k\cos\theta+\sin\theta)\,dx\,.$$

Since $x=R\theta$, where the angle θ goes from 0 to 45° (π/4 rad) and $dx=Rd\theta$. The work is then,

$$W=mgR\int_0^{\pi/4}(\mu_k\cos\theta+\sin\theta)\,d\theta=mgR(\mu_k\sin\theta-\cos\theta)\Big|_0^{\pi/4}$$

$$=mgR(\mu_k\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}+1)=\underline{mgR\left[1+\frac{\sqrt{2}}{2}(\mu_k-1)\right]}.$$

Now, consider the same sled pulled up a 22.5°-incline to a height $h=R(1-\frac{\sqrt{2}}{2})$. In this case, the distance up the incline is $x=R(1-\frac{\sqrt{2}}{2})/(\sin 22.5°)$. The work done in this case is then,

$$W=\int_a^b F(x)\,dx=\int_0^x mg(\mu_k\cos\theta+\sin\theta)=mg(\mu_k\cos\theta+\sin\theta)\int_0^x dx$$

$$=mg(\mu_k\cos\theta+\sin\theta)\frac{R(1-\frac{\sqrt{2}}{2})}{\sin\theta}=\underline{mgR(\mu_k\cot 22.5°+1)(1-\tfrac{\sqrt{2}}{2})}.$$

Comparing the two expressions for the work done, we find they are nearly identical.

†7-33. (a) When the two atoms are close to each other, the force is repulsive. At longer distances, the force is attractive; and at the equilibrium separation, the force is zero. Setting the force equal to zero and solving for x, gives

$$F = Ax^{-13} - Bx^{-7} = 0$$

$$\underline{x = \sqrt[6]{\frac{A}{B}}}.$$

(b) The work done is equal to the

$$W = \int_{x_{\text{eq}}}^{\infty} F_x dx = \int_{x_{\text{eq}}}^{\infty} Ax^{-13} - Bx^{-7} dx = A\left(\frac{x^{-12}}{12}\right) - B\left(\frac{x^{-6}}{6}\right)\Bigg|_{x_{\text{eq}}}^{\infty} = \frac{B^2}{12A} - \frac{B^2}{6A} = \underline{-\frac{B^2}{12A}}.$$

7-34. $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.060\text{ kg})\left[\left(\frac{160\text{ km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\right]^2 = \underline{59\text{ J}}$

†7-35. The Earth-Sun distance is $R = 1.5 \times 10^{11}$ m and the orbit is approximately circular, so the circumference of the Earth's orbit is $C = 2\pi R = 9.4 \times 10^{11}$ m. The Earth travels this distance in one year = 3.156×10^7 s, so the average speed of the Earth is $v = C/t = 3.0 \times 10^4$ m/s. The Earth's mass is 5.98×10^{24} kg. The kinetic energy of the Earth in its orbit is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(5.98 \times 10^{24}\text{ kg})\left(3.0 \times 10^4\text{ m/s}\right)^2 = \underline{2.7 \times 10^{33}\text{ J}}.$$

7-36. $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.11 \times 10^{-31}\text{ kg})\left(2.2 \times 10^6\text{ m/s}\right)^2 = \underline{2.2 \times 10^{-18}\text{ J}}$

7-37. $K_{\text{Skier}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})\left[\left(\frac{212.52\text{ km}}{1\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\right]^2 = \underline{1.3 \times 10^5\text{ J}}$

$$K_{\text{Runner}} = \frac{1}{2}mv^2 = \frac{1}{2}(75\text{ kg})\left[\left(\frac{44.88\text{ km}}{1\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\right]^2 = \underline{5.8 \times 10^3\text{ J}}$$

$$\frac{K_{\text{Skier}}}{K_{\text{Runner}}} = \frac{1.3 \times 10^5\text{ J}}{5.8 \times 10^3\text{ J}} = \underline{22}$$

7-38. $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1770\text{ kg})(120\text{ m/s})^2 = \underline{1.3 \times 10^7\text{ J}}$

$$m_{\text{TNT}} = 1.3 \times 10^7\text{ J}\left(\frac{1\text{ kg TNT}}{4.6 \times 10^6\text{ J}}\right) = \underline{2.8\text{ kg}}$$

†7-39. (a) 80 km/hr = 22.2 m/s, so $K = \tfrac{1}{2}mv^2 = \frac{1}{2}(1600\text{ kg})\left(22.2\text{ m/s}\right)^2 = \underline{4.0 \times 10^5\text{ J}}$

(b) 20 km/hr = 5.56 m/s, so $K = \tfrac{1}{2}mv^2 = \frac{1}{2}(1600\text{ kg})\left(5.56\text{ m/s}\right)^2 = \underline{2.5 \times 10^4\text{ J}}$

(c) 140 km/hr = 38.9 m/s, so $K = \tfrac{1}{2}mv^2 = \frac{1}{2}(1600\text{ kg})\left(38.9\text{ m/s}\right)^2 = \underline{1.2 \times 10^6\text{ J}}$

7-40. At 95 km/h, the probability $P_{95} = 3\%\left(\frac{95\text{ km/h}}{80\text{ km/h}}\right) = \underline{4.2\%}$.

Similarly, $P_{110} = 3\%\left(\frac{110\text{ km/h}}{80\text{ km/h}}\right) = \underline{5.7\%}$ and $P_{125} = 3\%\left(\frac{125\text{ km/h}}{80\text{ km/h}}\right) = \underline{7.3\%}$.

7-41. $K_0 = \frac{1}{2}mv^2 = \frac{1}{2}(100\ \text{lb})\left(\frac{1\ \text{kg}}{2.205\ \text{lb}}\right)(657\ \text{m/s})^2 = 9.8\times10^6\ \text{J}$

$K_{3\,\text{s}} = \frac{1}{2}mv^2 = \frac{1}{2}(100\ \text{lb})\left(\frac{1\ \text{kg}}{2.205\ \text{lb}}\right)(502\ \text{m/s})^2 = 5.7\times10^6\ \text{J}$

$\Delta K = \underline{-4.1\times10^6\ \text{J}}$

7-42. $K_{\text{Bullet}} = \frac{1}{2}m_{\text{Bullet}}v_{\text{Bullet}}^2 = \frac{1}{2}(0.015\ \text{kg})(630\ \text{m/s})^2 = \underline{3.0\times10^3\ \text{J}}$

$K_{\text{Ball}} = \frac{1}{2}m_{\text{Ball}}v_{\text{Ball}}^2 = \frac{1}{2}(15\ \text{kg})(6.3\ \text{m/s})^2 = \underline{3.0\times10^2\ \text{J}}$

The kinetic energy of the bullet is ten times greater than that of the bowling ball.

†7-43. $K_{\text{Ball}} = \frac{1}{2}m_{\text{Ball}}v_{\text{Ball}}^2 = \frac{1}{2}(0.045\ \text{kg})(45\ \text{m/s})^2 = \underline{46\ \text{J}}$

$K_{\text{Person}} = \frac{1}{2}m_{\text{Person}}v_{\text{Person}}^2 = \frac{1}{2}(75\ \text{kg})(1.0\ \text{m/s})^2 = \underline{38\ \text{J}}$

The kinetic energies of the golf ball and person are on the same order of magnitude.

7-44. The work done on the ball by the spring will be equal to the stored potential energy of the compressed spring. $W = \frac{1}{2}kx^2 = \frac{1}{2}(30\ \text{N/m})(0.080\ \text{m})^2 = \underline{0.096\ \text{J}}$

Assuming all of the spring potential energy is used to launch the ball, the kinetic energy will equal the work done on the ball, $\underline{0.096\ \text{J}}$.

The initial speed of the ball when it is launched will be $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.096\ \text{J})}{0.020\ \text{kg}}} = \underline{3.1\ \text{m/s}}$.

†7-45. The kinetic energy of the object as it is launched is $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150\ \text{kg})(5.0\ \text{m/s})^2 = \underline{1.9\ \text{J}}$.

The work done on the object is equal to the change in kinetic energy of the object. Since the object started from rest, the work is

$W = \Delta K = \underline{1.9\ \text{J}}$.

This energy was also the stored energy in the compressed spring. Therefore,

$x = \sqrt{\frac{2W}{k}} = \sqrt{\frac{2(1.9\ \text{J})}{20\ \text{N/m}}} = \underline{0.44\ \text{m}}$.

7-46. Assuming no other forces are acting on the player,

$\Delta K = W = F\Delta x = (500\ \text{N})(0.50\ \text{m}) = \underline{250\ \text{J}}$ and $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(250\ \text{J})}{60\ \text{kg}}} = \underline{2.9\ \text{m/s}}$.

7-47. $K = \frac{1}{2}mv^2 = \frac{1}{2}(1300\ \text{kg})(3100\ \text{m/s})^2 = \underline{6.2\times10^9\ \text{J}}$

7-48. Assume the initial potential energy of the stone is zero at the point it is released. The initial total mechanical energy is equal to the initial kinetic energy, $K(0)$. All of this energy becomes potential energy at the maximum height, h. Therefore, $K(0) = mgh$. At some intermediate point y, the stone has one-half its initial kinetic energy plus a potential energy that must be one-half its final potential energy. The total mechanical energy is

$K(0) = K(y) + mgy = mgh$

Therefore, $mgy + mg\frac{h}{2} = mgh$ which gives $\underline{y = \frac{h}{2}}$

†7-49. Let d be the depth the bullet penetrates when it is fired with a speed v. Since d is assumed to be proportional to the kinetic energy, we may write the relationship as $d = kv^2$, where k is some constant. The two cases may then be compared to find the speed of the second bullet.

$d_1 = kv_1^2$ or $0.8 \text{ cm} = k(160 \text{ m/s})$

$d_2 = kv_2^2$ or $1.2 \text{ cm} = k\,v_2^2$

Taking the ratio of these two equations yields, $v_2 = 160\sqrt{\dfrac{1.2}{0.9}} = \underline{196 \text{ m/s}}$.

7-50. (a) $W = \int_{x_1}^{x_2} F_x \; dx = \int_{x_1}^{x_2} -ax + bx^3 \; dx$

$$= -a\int_{x_1}^{x_2} x \; dx + b\int_{x_1}^{x_2} x^3 \; dx = \underline{-\tfrac{1}{2}a\left(x_2^2 - x_1^2\right) + \tfrac{1}{4}b\left(x_2^4 - x_1^4\right)}$$

(b) The work found in (a) is equal to the change in kinetic energy, ΔK.

†7-51. The stored potential energy in the spring is equal to the work done in moving it to its initial position, $W = \frac{1}{2}kx_1^2$. After the cantilever is released, the cantilever has both kinetic energy and spring potential energy, the sum of which equals the work done. Therefore,

$\frac{1}{2}kx_1^2 = K + \frac{1}{2}kx_2^2$

$$K = \tfrac{1}{2}k(x_1^2 - x_2^2) = \tfrac{1}{2}(2.5 \times 10^{-2} \text{ N/m})[(3.0 \times 10^{-8} \text{ m})^2 - (2.5 \times 10^{-8} \text{ m})^2]$$

$$= \underline{3.4 \times 10^{-18} \text{ J}}$$

7-52. (a) $F = |ma| = |(1500 \text{ kg})(-8.0 \text{ m/s}^2)| = \underline{1.2 \times 10^3 \text{ N}}$

(b) Using 90 km/h = 25 m/s and applying the kinematic formula, $a(x - x_0) = \frac{1}{2}(v^2 - v_0^2)$,

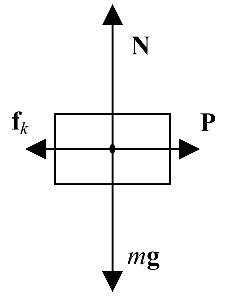
$$x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (25 \text{ m/s})^2}{2(-8.0 \text{ m/s}^2)} = \underline{39 \text{ m}}$$

(c) Assuming the braking force is constant, the work is

$W = Fx\cos 180° = -(1200 \text{ N})(39 \text{ m}) = \underline{-4.7 \times 10^4 \text{ J}}.$

(d) The change in the kinetic energy is the same as the work done by the braking force,

$\Delta K = W = \underline{-4.7 \times 10^4 \text{ J}}.$



†7-53. (a) $\Delta K = K_2 - K_1 = ½ (40 \text{ kg})(2.0 \text{ m/s})^2 = \underline{80 \text{ J}}$

(b) $W_f = \mathbf{f}_k \bullet \mathbf{x} = -f_k x = -\mu_k N x$

Applying Newton's second law in the vertical direction, $N = mg$. The sum of the horizontal forces is $P - f_k = ma$ or

$$a = \frac{P - \mu_k mg}{m} = \frac{250 \text{ N} - (0.60)(40 \text{ kg})(9.81 \text{ m/s}^2)}{(40 \text{ kg})} = 0.36 \text{ m/s}^2.$$

We can find the sliding distance from $x = \dfrac{v^2 - v_0^2}{2a} = \dfrac{(2 \text{ m/s})^2 - 0}{2(0.36 \text{ m/s}^2)} = 5.5 \text{ m}.$

The work done by the friction force is

$W_f = -\mu_k N x = -\mu_k mgx = -(0.60)(40 \text{ kg})(9.81 \text{ m/s}^2)(5.5 \text{ m}) = \underline{-1295 \text{ J}}.$

(c) $W_w = \mathbf{P} \bullet \mathbf{x} = Px = (250 \text{ N})(5.5 \text{ m}) = \underline{1375 \text{ J}}.$

Note that the total work done on box is $W = W_w + W_f = 80 \text{ J} = \Delta K$.

7-54. $W = \Delta U = mg\Delta y = (3200 \text{ kg})(9.81 \text{ m/s}^2)(6.0 \text{ m}) = \underline{1.9 \times 10^5 \text{ J}}$

†7-55. $W = mg\Delta y = (75 \text{ kg})(9.81 \text{ m/s}^2)(10 \text{ m}) = \underline{7.4 \times 10^3 \text{ J}}$

The metabolization of one apple is 4.6×10^5 J, so in climbing the stairs $(7.4 \times 10^3 \text{ J})/(4.6 \times 10^5 \text{ J}) = \underline{1.6\ \%}$ of the energy stored in an apple is used.

7-56. $v = \left(\dfrac{30\text{ km}}{1\text{ h}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{1000\text{ m}}{1\text{ km}}\right) = 8.3\text{ m/s}$

$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(6.0\text{ kg})(8.3\text{ m/s})^2 = \underline{210\text{ J}}$

$U = mgh = (6.0\text{ kg})(9.81\text{ m/s}^2)(90\text{ m}) = \underline{5300\text{ J}}$

†7-57. The gravitational potential energy of the water is $U = mgh$. The mass of the water can be determined from its density $\rho = m/V = 1000\text{ kg/m}^3$. To determine the volume, combine these two relations,

$$V = \frac{m}{\rho} = \frac{U}{\rho g h} = \frac{2.0\times10^{13}\text{ J}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(250\text{ m})} = \underline{8.2\times10^{6}\text{ m}^3}$$

7-58. The car has a total energy $E = K + U$. Since frictional effects are ignored, the energy of the car will be conserved. So, at the bottom of the hill, the car will have the same total energy as it had when it was released. Because the reference height is arbitrary in the determination of gravitational potential energy, it is convenient to choose the bottom of the hill as $h = 0$ m. The speed of the car when it is at a height of 60 m and heading downward is 13 km/h = 3.6 m/s.

$$\underbrace{\tfrac{1}{2}mv_1^2 + mgh_1}_{\text{total energy at release}} = \underbrace{\tfrac{1}{2}mv_2^2 + mgh_2}_{\text{total energy at bottom}}$$

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2$$

Divide through by the mass m and solve for the speed v_2.

$$v_2 = \sqrt{v_1^2 + 2gh_1} = \sqrt{(3.6\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(60\text{ m})} = \underline{34.5\text{ m/s or }124\text{ km/h}}$$

†7-59. Applying the principle of the conservation of mechanical energy to this situation

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$\tfrac{1}{2}mv_1^2 = mgh_2$$

$$h_2 = \frac{v_1^2}{2g} = \frac{(10\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \underline{5.1\text{ m}}$$

However, the athlete actually reaches a height of 5.7 m. In the process of performing the vault, the athlete exerts an additional amount of internal energy in pulling himself upward that results in the additional 0.6 m height.

7-60. $\underbrace{\tfrac{1}{2}mv_1^2 + mgh_1}_{\text{total energy at top}} = \underbrace{\tfrac{1}{2}mv_2^2 + mgh_2}_{\text{total energy at bottom}}$

$$mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gh_1} = \sqrt{2(9.81\text{ m/s}^2)(45\text{ m})} = \underline{29.7\text{ m/s}}$$

†7-61. Use conservation of energy, since friction effects are negligible.

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$mgh_1 = \tfrac{1}{2}mv_2^2$ which gives $v_2 = \sqrt{2gh_1} = \sqrt{2(9.81\text{ m/s}^2)(500\text{ m})} = \underline{99\text{ m/s}}$

The kinetic energy of the 2.0×10^7 kg of snow is

$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.0\times10^7\text{ kg})(99\text{ m/s})^2 = \underline{9.8\times10^{10}\text{ J}}$.

The equivalent amount of TNT is

$N = \dfrac{9.8\times10^{10}\text{ J}}{4.2\times10^{9}\text{ J/ton}} = \underline{23\text{ tons of TNT}}$.

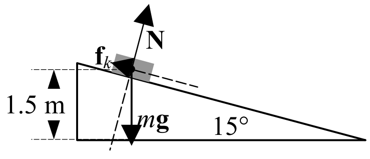
7-62. The difference in her total energy between her jump and her landing is equal to the amount of work done by frictional forces, $W = \Delta E$. Therefore,

$$W = E_2 - E_1 = (K_2 + U_2) - (K_1 + U_1)$$

On the aircraft flying parallel to the ground below, she has a velocity equal to that of the aircraft and a corresponding kinetic energy. However, we are interested in her speed in the vertical direction, so we'll assume her kinetic energy as she leaves the airplane is $K_1 = 0$ J. Choose the ground as our potential energy reference, $U_2 = 0$ J.

$$W = K_2 - U_1 = \tfrac{1}{2}mv^2 - mgh = \tfrac{1}{2}(60\text{ kg})(5.0\text{ m/s})^2 - (60\text{ kg})(9.81\text{ m/s}^2)(800\text{ m}) = \underline{-4.7\times 10^5\text{ J}}$$

†7-63. Consider the free-body diagram superposed on the drawing. In the direction perpendicular to the inclined plane, $N - mg\cos\theta = 0$ or $N = mg\cos\theta$. The distance the block slides is $s = (1.5\text{ m})/\sin(15°) = 5.8$ m; and the work done by the frictional force on the block is



$$W = E_2 - E_1 = (K_2 + U_2) - (K_1 + U_1)$$
$$= K_2 - U_1 = -fs$$

which is then used to find the coefficient of kinetic friction, since $f_k = \mu_k N$.

$$mgh_1 - \tfrac{1}{2}mv_2^2 = \mu_k Ns$$

$$\mu_k = \frac{mgh_1 - \frac{1}{2}mv_2^2}{Ns} = \frac{mgh_1 - \frac{1}{2}mv_2^2}{(mg\cos\theta)s} = \frac{2gh_1 - v_2^2}{2(g\cos\theta)s} = \frac{2(9.81\text{ m/s}^2)(1.5\text{ m}) - (3.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)(\cos 15^\circ)(5.8\text{ m})} = \underline{0.16}$$

7-64.
$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$
$$mgh_1 = \tfrac{1}{2}mv_2^2$$
$$v_2 = \sqrt{2gh_1} = \sqrt{2(9.81\text{ m/s}^2)(148\text{ m})} = \underline{53.9\text{ m/s}}$$

†7-65. The difference in the total energy between the release from rest of the ball and its landing is equal to the amount of work done by frictional forces, $W = \Delta E = (K_2 - U_2) - (K_1 - U_1)$. The fraction lost to air friction is equal to W/U_1.

$$W = K_2 - U_1 = \tfrac{1}{2}mv_2^2 - mgh_1$$

$$\frac{W}{U_1} = \left|\frac{\frac{1}{2}mv_2^2 - mgh_1}{mgh_1}\right| = \left|\frac{v_2^2 - 2gh_1}{2gh_1}\right| = \left|\frac{(9.0\text{ m/s})^2 - 2(9.81\text{ m/s}^2)(20\text{ m})}{2(9.81\text{ m/s}^2)(20\text{ m})}\right| = \underline{0.79\text{ or }79\%}.$$

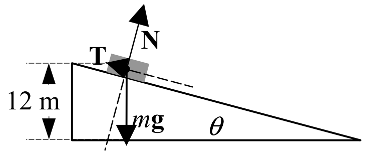
7-66. Since frictional effects are ignored, conservation of energy applies.

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$
$$mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$
$$h_2 = \frac{gh_1 - \frac{1}{2}v_2^2}{g} = \frac{(9.81\text{ m/s}^2)(45\text{ m}) - \frac{1}{2}(15\text{ m/s})^2}{(9.81\text{ m/s}^2)} = \underline{33.5\text{ m}}$$

†7-67. Since frictional effects are ignored, conservation of energy applies.

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$
$$mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$
$$v_2 = \sqrt{2g\left(h_1 - h_2\right)} = \sqrt{2(9.81\text{ m/s}^2)(3.0\text{ m})} = \underline{7.7\text{ m/s}}$$

7-68. (a) Consider the free-body diagram superposed on the drawing. Ignoring frictional forces, in the direction parallel to the incline, the sum of the forces is $T - mg\sin\theta = 0$. Since the distances are given, the angle $\theta = \sin^{-1}(12\text{ m}/500\text{ m}) = 1.38°$. Each metric ton is 1000 kg.



$$T = mg\sin\theta = (70\text{ t} + 35\text{ t})\left(\frac{1000\text{ kg}}{1\text{ t}}\right)(9.81\text{ m/s}^2)\sin(1.38°) = \underline{2.5\times10^4\text{ N}}$$

(b) The work done is equal to the change in the gravitational potential energy of the barge and carriage, $W = \Delta U = mg(h_2 - h_1) = (1.05\times10^5\text{ kg})(9.81\text{ m/s}^2)(12\text{ m}) = \underline{1.2\times10^7\text{ J}}$.

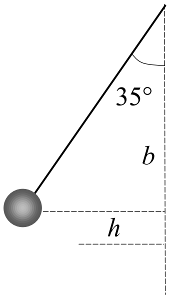
(c) Using conservation of energy, $\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

$$mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.81\text{ m/s}^2)(12\text{ m})} = \underline{15\text{ m/s}}$$

†7-69. When the ball is displaced from its equilibrium position, it is raised to a height h relative to its lowest position before it is released from rest. To find the height h, a horizontal line is drawn from the ball's initial position to the vertical line. A right triangle is then formed, from which we can find the length of the side labeled b. The initial height is then, $h = (10\text{ m}) - b = (10\text{ m}) - (10\text{ m})\cos 35° = (10\text{ m})(1 - \cos 35°)$.

We can then apply the principle of conservation of mechanical energy to find the ball's kinetic energy at its lowest point where its potential energy $U_2 = 0$.



$$K_2 + U_2 = K_1 + U_1$$

$$K_2 = U_1$$

$$K_2 = mgh_1 = (600\text{ kg})(9.81\text{ m/s}^2)(10\text{ m})(1 - \cos 35°) = \underline{1.1\times10^4\text{ J}}$$

7-70. When the stone is launched, it has a total energy $E_1 = K_1 + U_1 = K_1 = \frac{1}{2}mv_1^2$. If frictional forces do work W_1 on the stone as it flies upward, then the total energy is reduced to a new value $E_2 = K_2 + U_2 = mgh_2 = E_1 + W_1 = \frac{1}{2}mv_1^2 + W_1 = \frac{1}{2}mv_1^2 - fh_2$ (the maximum height the stone reaches is less than it would have attained without air friction). In the downward portion of the journey, the energy of the stone is further reduced by frictional forces and the stone reaches the ground with a total energy $E_3 = K_3 + U_3 = K_3 = \frac{1}{2}mv_1^2 - 2fh_2$. As a result the landing speed $v_3 < v_1$. To understand the time difference, we need to determine the net acceleration during each part of the flight. During the upward portion, we have

$$mgh_2 = \tfrac{1}{2}mv_1^2 - fh_2 \text{ or } v_1^2 = 2gh_2 - \frac{2f}{m}h_2 = 2h_2\left(g - \frac{f}{m}\right)$$

From kinematics, for constant acceleration, we have the relation, $a = \dfrac{v_2^2 - v_1^2}{2y}$.

So, during the upward portion of the flight, the acceleration is $a = \dfrac{-v_1^2}{2h_2}$. The net acceleration is negative since the frictional force and gravitational are both directed downward as the stone moves upward. The time for the upward flight can be found from another kinematic relation,

$$y = \tfrac{1}{2}at^2 \text{ or, in this case, } t_1 = \sqrt{\frac{2h_2}{a}} = \sqrt{\frac{4h_2^2}{v_1^2}} = \sqrt{\frac{4h_2^2}{2h_2\left(g - \frac{f}{m}\right)}} = \sqrt{\frac{2h_2}{\left(g - \frac{f}{m}\right)}}.$$

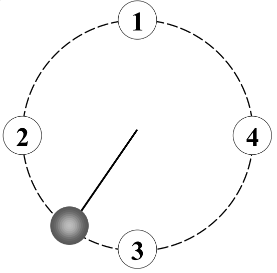
The ratio of *f*/*m* will always be less than *g*. Applying a similar analysis to the downward portion of the flight,

$$v_3^2 = 2gh_2 - \frac{4f}{m}h_2 = 2h_2\left(g - \frac{2f}{m}\right)$$

$$\text{and } t_2 = \sqrt{\frac{2h_2}{a}} = \sqrt{\frac{4h_2^2}{v_3^2}} = \sqrt{\frac{4h_2^2}{2h_2\left(g - \frac{2f}{m}\right)}} = \sqrt{\frac{2h_2}{\left(g - \frac{2f}{m}\right)}}$$

To see that $t_2 > t_1$, note that the denominator under the radical for t_1 is larger than that for t_2.

†7-71. At position **1**, choosing the downward direction as positive, the sum of the forces on the stone is



$$T_1 + mg = \frac{mv_1^2}{r}$$

Since the tension is about zero, the speed at the top of the circle is $v_1^2 = rg$. Therefore, the total energy is

$$E = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mrg + 2mrg = \tfrac{5}{2}mrg$$

At position **3**, the tension is directed upward and the sum of the forces on the stone is

$T_3 - mg = \dfrac{mv_3^2}{r}$. To find v_3, consider the total energy at position **3**.

$$E = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mgh_3 = \tfrac{1}{2}mv_3^2 = \tfrac{5}{2}mrg$$

$$v_3^2 = 5rg$$

Therefore, the tension at position **3** is

$$T_3 = mg + \frac{mv_3^2}{r} = mg + \frac{m5rg}{r} = 6mg = 6(0.90\text{ kg})(9.81\text{ m/s}^2) = \underline{53\text{ N}}.$$

7-72. At its highest point, the velocity of the ball is in the horizontal direction and has the same magnitude as the horizontal component of the initial velocity, $v = v_0\cos\theta$. So, the kinetic energy at the highest point is

$$K_\text{f} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(v_0\cos\theta)^2 = \tfrac{1}{2}(0.17\text{ kg})(28\text{ m/s}\cos 30°)^2 = \underline{50\text{ J}}.$$

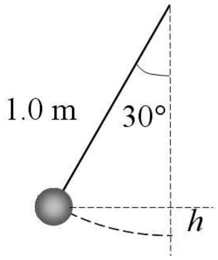
The final potential energy can be found using conservation of energy. The initial kinetic energy is

$$K_\text{i} = \tfrac{1}{2}mv_0{}^2 = \tfrac{1}{2}(0.17\text{ kg})(28\text{ m/s})^2 = 67\text{ J}.$$

$$P_\text{f} = \Delta K = 67\text{ J} - 50\text{ J} = \underline{17\text{ J}}$$

7-73. $v_\text{top} = \sqrt{v_\text{bottom}^2 - 2gh} = \sqrt{(172\text{ m/s})_\text{bottom}^2 - 2(9.81\text{ m/s}^2)(1000\text{ m})} = \underline{1.0\times 10^2\text{ m/s or }360\text{ km/h}}$

7-74. At any point along its trajectory, the pendulum bob will have the same total energy as it has at its starting position. The height above the lowest position is $h = L(1 - \cos 30°)$, where *L* is the pendulum length. For a given angle $\theta < 30°$, the total energy is



$mgh = \frac{1}{2}mv^2 + mg(1-\cos\theta)L.$

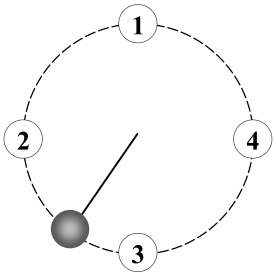
At the lowest point, $\theta = 0°$ and the speed is

$v_{\text{bottom}} = \sqrt{2g(\cos\theta - \cos 30°)} = \sqrt{2(9.81\text{ m/s}^2)(1-\cos 30°)} = \underline{1.6\text{ m/s}}.$

The angle at which the speed is one-half this value is

$$2g(\cos\theta - \cos 30°) = v^2 = \frac{v_{\text{bottom}}^2}{4}.$$

$$\theta = \cos^{-1}\left(\frac{v_{\text{bottom}}^2}{8\text{g}} + \cos 30°\right) = \cos^{-1}\left(\frac{(1.6\text{ m/s})^2}{8(9.81\text{ m/s}^2)} + \cos 30°\right) = \underline{26°}$$



7-75. At position **1**, choosing the downward direction as positive, the sum of the forces on the stone is

$$T_1 + mg = \frac{mv_1^2}{r}$$

Since the tension is about zero, the speed at the top of the circle is $v_1^2 = gr$. Therefore, the total energy is

$$E = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mgr + 2mgr = \tfrac{5}{2}mgr$$

At position **3**, the tension is directed upward and the sum of the forces on the stone is

$T_3 - mg = \dfrac{mv_3^2}{r}$. To find v_3, consider the total energy at position **3**.

$$E = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mgh_3 = \tfrac{1}{2}mv_3^2 = \tfrac{5}{2}mgr$$

$v_3^2 = 5gr$ or $v_3 = \underline{\sqrt{5gr}}$

7-76. At the top of the loop, the net force acting on a car is equal to N + mg and both forces are directed downward. This force is the cetripetal force, $F = mv^2/R$, where R is the radius of the loop. For the car to stay on the track, the normal force $N \geq 0$ N. The minimum speed that the car must be traveling to stay on the track of the top of the loop occurs when $N = 0$ N.

$$\frac{mv^2}{R} = N - mg = mg \quad \text{or} \quad v = \sqrt{Rg} = \sqrt{(10\text{ m})(9.81\text{ m/s}^2)} = \underline{9.9\text{ m/s}}$$

To find the minimum speed at the bottom, apply conservation of energy.

$$K_2 + U_2 = K_1 + U_1$$

$$\tfrac{1}{2}mv_2^2 + mgh_2 = \tfrac{1}{2}mv_1^2 + mgh_1$$

$$\tfrac{1}{2}mv_2^2 + mgh_2 = \tfrac{1}{2}mv_1^2$$

Then, solve for v_1.

$$v_2^2 + 2gh_2 = v_1^2$$

$$v_1 = \sqrt{v_2^2 + 2gh_2} = \sqrt{(9.9\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(40\text{ m})} = \underline{29.7\text{ m/s}}$$

7-77. (a) $\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$

$$mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gh_1} = \sqrt{2(9.81\text{ m/s}^2)(30\text{ m})} = \underline{24\text{ m/s}}$$

(b) At the bottom of the circular valley, the sum of the vertical forces on the car is

$$N - mg = \frac{mv_2^2}{R} = ma$$

The passengers feel an acceleration of 8g. Therefore,

$$\frac{v_2^2}{R} = 8g \quad \text{and} \quad R = \frac{v_2^2}{8g} = \frac{(24\ \text{m/s})^2}{8(9.81\ \text{m/s}^2)} = \underline{7.3\ \text{m}}$$

(c) At the top of the hill, $N + mg = \dfrac{mv_3^2}{R}$

If the passengers feel a normal force of zero N, then, $v_3 = \sqrt{gR}$

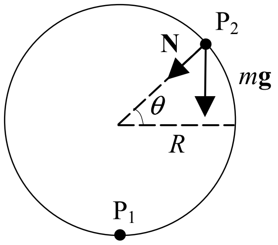
To find the height h', reapply the conservation of energy.

$$\tfrac{1}{2}mv_2^2 + mgh_2 = \tfrac{1}{2}mv_3^2 + mgh'$$

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}mv_3^2 + mgh'$$

$$h' = \frac{v_2^2 - v_3^2}{2g} = \frac{v_2^2 - gR}{2g} = \frac{(24\ \text{m/s})^2 - (9.81\ \text{m/s}^2)(7.3\ \text{m})}{2(9.81\ \text{m/s}^2)} = \underline{25.7\ \text{m}}$$

7-78. The initial speed at the bottom of the circle at point P_1 is $v_1 = (2gr)^{1/2}$. The cart moves along the circle to the point P_2 where the normal force goes to zero N. At the position P_2, the sum of the forces in the radial direction is $N + mg\sin\theta = mv_2^2/R$. To find the angle θ, the height is found using $h = R(1 + \sin\theta)$. Begin by applying conservation of energy,



$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_2^2 + mgh$$

$$mv_2^2 = mv_1^2 - 2mgh = m\left(2\sqrt{gR}\right)^2 - 2mgR(1+\sin\theta) = 2mgR(1-\sin\theta)$$

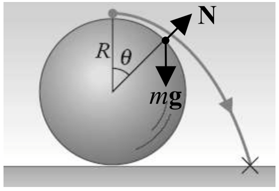
This relationship can be substituted into the centripetal force equation.

$$mg\sin\theta = \frac{mv_2^2}{R} = \frac{2mgR(1-\sin\theta)}{R}$$

$$\sin\theta = 2(1-\sin\theta)$$

$$\theta = \sin^{-1}\left(\tfrac{2}{3}\right) = \underline{42^\circ}$$

†7-79. The particle will slide along the sphere until the normal force N goes to zero. At the angular position shown, the sum of the forces on the particle in the radial direction is $mg\cos\theta - N = mv^2/R$, which gives $v^2 = gR\cos\theta$. The conservation of energy is then applied to the situation,



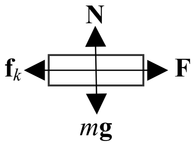
$$mgR = mgR\cos\theta + \tfrac{1}{2}mv^2 = mgR\cos\theta + \tfrac{1}{2}m(gR\cos\theta)..$$

Solving for θ gives, $\theta = \cos^{-1}\left(\tfrac{2}{3}\right) = \underline{48.2^\circ}$.

7-80. $W = \mathbf{F} \bullet \mathbf{y} = mgh = (0.20\ \text{kg})(9.81\ \text{m/s}^2)(35\ \text{m}) = \underline{69\ \text{J}}$

7-81. $W = \mathbf{F} \bullet \mathbf{s} = (150\ \text{N})(20\ \text{m})\cos 45^\circ = \underline{2100\ \text{J}}$

7-82. The sum of the forces on the crate in the vertical direction is $N - mg = 0$, so $N = mg$. In the horizontal direction, the sum of the forces is $F - f_k = 0$, so $F = f_k = \mu_k N = \mu_k mg$. The crate is pushed for a distance $s = 15$ m, so the work done by the man is $W = \mathbf{F} \bullet \mathbf{s} = \mu_k mgs = (0.50)(120\ \text{kg})(9.81\ \text{m/s}^2)(15\ \text{m}) = \underline{8800\ \text{J}}$.



7-83. In the first case, the automobile accelerates from 20 to 25 m/s.

$$W = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}m(v_2^2 - v_1^2) = \tfrac{1}{2}(1500\text{ kg})\left[(25\text{ m/s})^2 - (20\text{ m/s})^2\right] = \underline{1.69 \times 10^5\text{ J}}$$

In the second case, the automobile accelerates from 25 to 30 m/s.

$$W = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}m(v_2^2 - v_1^2) = \tfrac{1}{2}(1500\text{ kg})\left[(30\text{ m/s})^2 - (25\text{ m/s})^2\right] = \underline{2.06 \times 10^5\text{ J}}$$

7-84. In raising the books, she applies an upward force F. The books are assumed to move upward at a constant speed, so the sum of the vertical forces equals $F - mg = 0$ and $F = mg$. The work done by the woman on the books is

$W_{\text{up}} = \mathbf{F} \bullet \mathbf{s} = Fs\cos\theta = Fs = mgs = (20\text{ kg})(9.81\text{ m/s}^2)(1.8\text{ m}) = \underline{+350\text{ J}}$.

When the books are lowered at a constant rate, the same analysis applies. However, in this case the force continues to be applied upward and the displacement is downward. Therefore,

$W_{\text{down}} = \mathbf{F} \bullet \mathbf{s} = Fs\cos\theta = -Fs = -mgs = -(20\text{ kg})(9.81\text{ m/s}^2)(1.8\text{ m}) = \underline{-350\text{ J}}$.

The total work done is

$W = W_{\text{up}} + W_{\text{down}} = +350\text{ J} - 350\text{ J} = \underline{0\text{ J}}$.

While the woman's muscles are causing work to be done in the upward and downward motions, there is nothing in the analysis to indicate what is happening internally. It will depend upon the details of how the woman does the work over time.

†7-85. (a) Frictional forces do work on the car over the stopping distance, $W = -fs$. This results in a change in kinetic energy ΔK.

$$W = -\mu mgs = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$s = \frac{0 - \tfrac{1}{2}mv_1^2}{-\mu mg} = \frac{\tfrac{1}{2}(25\text{ m/s})^2}{(0.90)(9.81\text{ m/s}^2)} = \underline{35.4\text{ m}}$$

To determine the acceleration, assuming the acceleration is constant as the car is slowing, we can use the kinematic relation

$$v_2^2 - v_1^2 = 2as.$$

$$a = \frac{v_2^2 - v_1^2}{2s} = \frac{0 - (25\text{ m/s})^2}{2(35.4\text{ m})} = \underline{-8.8\text{ m/s}^2}$$

(b) $f = \mu N = \mu mg = (0.90)(1200\text{ kg})(9.81\text{ m/s}^2) = \underline{1.06 \times 10^4\text{ N}}$

$W = -fs = -(10\,600\text{ N})(35.4\text{ m}) = \underline{3.75 \times 10^5\text{ J}}$

7-86. (a) $K_1 = \tfrac{1}{2}mv_1^2 = mgh_1 = (0.050\text{ kg})(9.81\text{ m/s}^2)(1.5\text{ m}) = \underline{0.74\text{ J}}$

(b) $K_2 = \tfrac{1}{2}mv_2^2 = mgh_2 = (0.050\text{ kg})(9.81\text{ m/s}^2)(1.0\text{ m}) = \underline{0.49\text{ J}}$

(c) $K_1 - K_2 = 0.74\text{ J} - 0.49\text{ J} = \underline{0.25\text{ J}}$

7-87. (a) $U_1 = mgh = (1200\text{ kg})(9.81\text{ m/s}^2)(2000\text{ m}) = \underline{2.35 \times 10^7\text{ J}}$

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(1200\text{ kg})\left[\left(250\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\right]^2 = \underline{2.89 \times 10^6\text{ J}}$$

(b) $U_2 = mgh_2 = (1200\text{ kg})(9.81\text{ m/s}^2)(1500\text{ m}) = \underline{1.77 \times 10^7\text{ J}}$

$$\begin{aligned} K_2 &= K_1 + U_1 - U_2 \\ &= 2.86 \times 10^6\text{ J} + 2.35 \times 10^7\text{ J} - 1.77 \times 10^7\text{ J} = \underline{8.7 \times 10^6\text{ J}} \end{aligned}$$

$$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(8.7 \times 10^6\text{ J})}{1200\text{ kg}}} = \underline{120\text{ m/s or }430\text{ km/h}}$$

7-88. Assuming no losses due to frictional forces, apply conservation of energy to determine the kinetic energy at the bottom of valley and the corresponding speed.

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(30\text{ m})} = \underline{24\text{ m/s}}$$

The speed at the top of the second hill can be determined in a similar manner.

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_3^2 + mgh_3$$

$$mgh_1 = \tfrac{1}{2}mv_3^2 + mgh_3$$

$$v_3 = \sqrt{2g(h_1 - h_3)} = \sqrt{2(9.81\text{ m/s}^2)(10\text{ m})} = \underline{14\text{ m/s}}$$

7-89. (a) $W_{\text{pull}} = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta = Fs = (300\text{ N})(0.50\text{ m}) = \underline{150\text{ J}}$

(b) The bow exerts a constant force of 300 N on the arrow over the distance of 0.50 m. Assuming any other forces are negligible, the bow does the work that changes the kinetic energy of the arrow from zero J to the value it has at the release. $W = \Delta K = \underline{150\text{ J}}$

(c) $K = \tfrac{1}{2}mv_1^2$

$$v_1 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(150\text{ J})}{(0.020\text{ kg})}} = \underline{122\text{ m/s}}$$

(d) Assuming the arrow lands at the same height as it was launched and there are no losses due to air friction,

$$x_{\max} = \frac{v_1^2\sin 2\theta}{g} = \frac{(122\text{ m/s})^2\sin[2(45°)]}{(9.81\text{ m/s}^2)} = \underline{1520\text{ m}}$$

(e) Assuming the arrow lands at the same height as it was launched and there are no losses due to air friction, the kinetic energy it has when it lands will equal its kinetic energy at launch. Therefore, the landing speed will be 122 m/s.

7-90. (a) From the range, the initial speed and, thus, the initial kinetic energy can be calculated, assuming no losses due to air friction.

$$x_{\max} = \frac{v_0^2\sin 2\theta}{g}$$

$$v_0 = \sqrt{\frac{gx_{\max}}{\sin 2\theta}} = \sqrt{\frac{(9.81\text{ m/s}^2)(180\text{ m})}{\sin 90°}} = 42\text{ m/s}$$

$$K = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(77\text{ kg})(42\text{ m/s})^2 = \underline{6.8\times 10^4\text{ J}}$$

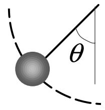
(b) At the highest point, the vertical component of the stone's velocity is equal to zero m/s. The kinetic energy is then,

$$K_{\text{top}} = \tfrac{1}{2}m(v_0\cos\theta)^2 = K(\cos 45°)^2 = \tfrac{1}{2}K = \underline{3.4\times 10^4\text{ J}}$$

†7-91. The energy lost to friction is the difference between the final and initial mechanical energy of the luger. The initial mechanical energy is equal to the potential energy because the luger starts from rest.

$$\begin{aligned}\Delta E &= mgh_1 - \left(\tfrac{1}{2}mv_2^2 + mgh_2\right)\\ &= mg(h_1 - h_2) - \tfrac{1}{2}mv_2^2\\ &= (95\text{ kg})(9.81\text{ m/s}^2)(350\text{ m} - 240\text{ m}) - \tfrac{1}{2}(95\text{ kg})\left[(130\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\right]^2\\ &= \underline{4.1\times 10^4\text{ J}}\end{aligned}$$

7-92. Before the release, the total energy of the pendulum bob is equal to its potential energy, *mgl*. At the bottom of the swing, conservation of energy will give the kinetic energy, and thus, the speed.



$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_2 = \underline{\sqrt{2gl}}$$

At the bottom of the swing, the sum of the vertical forces acting on the bob is

$$T - mg = \frac{mv_2^2}{r}$$

where *T* is the tension in the string.

$$T = \frac{mv_2^2}{r} + mg = m\left(\frac{v_2^2}{l} + g\right) = m\left(\frac{2gl}{l} + g\right) = \underline{3mg}$$

The tension in the string is three times greater when the pendulum is swinging from an initial angle $\theta = 90°$, than when it is simply hanging at rest its equilibrium position.

7-93. (a) $\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$

$$mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(34\text{ m})} = \underline{25.8\text{ m/s}}$$

(b) 100 km/h = 27.8 m/s

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2$$

$$v_1^2 = v_2^2 - 2gh_1$$

$$v_1 = \sqrt{v_2^2 - 2gh_1} = \sqrt{(27.8\text{ m/s})^2 - 2(9.81\text{ m/s}^2)(34\text{ m})} = \underline{10.3\text{ m/s}}$$

7-94. (a) $\tfrac{1}{2}mv_1^2 + mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$

$$mgh_1 = \tfrac{1}{2}mv_2^2 + mgh_2$$

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.81\text{ m/s}^2)(6.0\text{ m} - 1.0\text{ m})} = \underline{9.9\text{ m/s}}$$

(b) The horizontal distance is determined by the time she is in the air before hitting the water. The flight time is the time it takes to fall 1.0 m.

$$y = \tfrac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(1.0\text{ m})}{(9.81\text{ m/s}^2)}} = 0.45\text{ s}$$

$$x = v_0t = (9.9\text{ m/s})(0.45\text{ s}) = \underline{4.5\text{ m}}$$